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# Grey Elite Multi-parent Crossover Algorithm Based on High-dimensional Multi-objective Optimization Design

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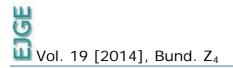
## ABSTRACT

The work proposed a new evolutionary algorithm based on high-dimensional multi-objective constraint optimization design of mixed discrete variables. The method applied relative degree of grey incidence to transform high-dimensional multi-objective optimization problem into single objective one. After that, the single objective problem was solved by elite parent crossover optimization algorithm. The dynamic penalty function and elite preservation strategy were introduced to add parent selection pressure and accelerate algorithm convergence. The work developed elite multi-parent crossover algorithm and elite multi-parent crossover optimization algorithm MOPTDEMPCOA based on high-dimensional multi-objective optimization problem of mixed discrete variables. The optimal example shows that this algorithm, with good global convergence and reliable operation, can be widely and effectively used to solve high-dimensional multi-objective optimization and robust design problems of mixed discrete variables.

KEYWORDS: Mixed Discrete Variables, Relative Degree of Grey Incidence, Nonlinear Constraint Optimization, Dynamic Penalty Function, Evolutionary Algorithm, Elite Preservation

# **INTRODUCTION**

In engineering optimization design, discontinuous variables contain integer (e.g., the number of teeth) or discrete ones (e.g., gear modulus). It has common significance to analyze the mixed discrete variable optimization design problems based on coexistence of integer, discrete and continuous variables. With some deficiency and difficulty, discrete variable optimization is one of the difficult fields of mathematical programming and operational research <sup>[1]</sup>. A basic principle of benefit maximization and cost minimization indicates that optimization design is multi-objective. In application, the incomparability and conflict of objective functions make it difficult to solve multi-objective optimization problem by classical optimization method <sup>[2]</sup>. Evolutionary algorithm, as a computing technique based on population operation, can parallel search multiple solutions in space and improve parallel solving ability by comparability among different solutions. It can be used to solve multi-objective optimization problems. Particle Swarm Optimization (PSO) is an evolutionary algorithm for optimization problem solution<sup>[3]</sup>. PSO, easy to be understood and implemented, has higher efficiency than Genetic Algorithm (GA). However, the optimal solution of complicated problems cannot be achieved based on premature phenomenon of PSO. The current research is to expand PSO application to multi-objective solution problems. PSO can only be used in optimization problems with two objective functions. Optimization problems with more than three objective functions are difficult to solve because of large amount of calculation<sup>[4]</sup>. Grey system theory, proposed by Professor Deng Julong, has been



widely used in fields for two decades <sup>[5]</sup>. Multi-objective optimization design methods proposed by Luo Youxin, including grey clustering method, grey correlation degree, grey decision-making method, are used to evaluate objectives <sup>[6]</sup>. Chen Yibao applied Deng correlation degree to design multi-objective optimization of beam pumping unit for optimal results <sup>[7]</sup>. In multi-objective optimization by Deng correlation degree, it is complicated to determine weight. While transforming multi-objective into single one by grey difference degree, the method may not change grey difference degree, with changed objective values. The global optimization solving ability and computing efficiency are to be improved. Multi-objective optimization design has problems to be solved [8-10], and elite multi-parent crossover algorithm based on multi-objective grey relative correlation of mixed discrete variables is seldom reported. Relative degree of grey incidence <sup>[11]</sup> is used to select global and individual extremes of high-dimensional multi-objective optimization design problems with mixed discrete variables. By improving elite multi-parent crossover algorithm, the work introduces dynamic penalty function to construct new fitness function. The work develops grey elite multi-parent crossover algorithm procedure based on mixed discrete variable optimization, to realize solution of high-dimensional multi-objective optimization problems. The example of engineering design indicates that this algorithm, with good global convergence and reliable operation, can be widely and effectively used in optimization design.

# OPTIMIZATION DESIGN METHOD BASED ON RELATIVE DEGREE OF GREY INCIDENCE

#### High-dimensional multi-objective optimization design model

The multi-objective non-linear problem is denoted as follows:

min 
$$F(\mathbf{x}) = \{f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_k(\mathbf{x})\}^T, (r = 1, 2, \dots, k)$$

S.t.

$$g_i(\mathbf{x}) \le 0, i = 1, 2, \cdots m \tag{1}$$
$$h_q(\mathbf{x}) = 0, q = 1, 2, \cdots, p$$

where design variables  $\mathbf{x} = [x_1, x_2, \dots, x_n]^T$ , and n is the number of design variables;  $f_r(\mathbf{x})(r = 1, 2, \dots, k)$  the rth objective function;  $g(\mathbf{x})$  inequality constraint;  $h(\mathbf{x})$  equality constraint.

## Relative degree solution of grey incidence based on multiobjective optimization design

The research subject of grey system theory is small sample and poor information uncertainty system with some information unknown. In grey system, dynamic development similarity of objects and factors is denoted as relative degree. The larger the relative degree is, the larger the similarity between objects is. Grey correlation degree analysis is a simple and special method overcoming deficiencies of traditional mathematical statistics including large sample requirement and calculation, as well as disagreement of quantitative and qualitative results. It is assumed that the reference vector sequence  $\mathbf{F}_{00} = (F_{001}, F_{002}, \dots, F_{00M}), M$  is the number of multi-objective functions; objective vector sequence  $\mathbf{F}_{01} = (F_{01}, F_{02}, \dots, F_{0M})$ .  $\mathbf{F}_{00}$  and  $\mathbf{F}_{01}$ are conducted with the initial image processing to achieve  $\mathbf{F}_0$  and  $\mathbf{F}$ .  $\mathbf{F}_0 = F_{00} / F_{00}(1), \mathbf{F} = F_{01} / F_{01}(1)$ .

The relative correlation degree <sup>[11]</sup> of  $\mathbf{F}_0$  and  $\mathbf{F}$  is expressed as:

$$\varepsilon = \frac{1 + |s_0| + |s|}{1 + |s_0| + |s| + |s_0 - s|}$$
where  $|s_0| = \left| \sum_{k=2}^{M} F_0^0(k) + \frac{1}{2} F_0^0(M) \right|$ ;  $|s| = \left| \sum_{k=2}^{M} F^0(k) + \frac{1}{2} F^0(M) \right|$ ;  
 $|s_0 - s| = \left| \sum_{k=2}^{M} (F_0^0(k) - F^0(k)) + \frac{1}{2} (F_0^0(M) - F^0(M)) \right|$ ;  
 $F_0^0(k) = F_0(k) - F_0(1)$   
 $F^0(k) = F(k) - F(1)(k = 1, 2, \dots, M)$ 
(2)

Relative degree of grey incidence has many properties. Compared with Deng grey correlation degree, relative degree of grey incidence will change as any datum in  $\mathbf{F}_0$  and  $\mathbf{F}$  changes, thus making a great significance in iteration of optimization design. After initial image processing, magnitude difference between  $\mathbf{F}_{00}$  and  $\mathbf{F}_{01}$  has little influence on optimization. In optimization design, grey difference degree is introduced to solve the minimum of objective function.

$$d = 1 - \varepsilon \tag{3}$$

Equation (3) shows that the smaller grey difference degree is, the larger the similarity between vector sequences  $\mathbf{F}_{01}$  and  $\mathbf{F}_{00}$  is. The optimal result is closer to the ideal value. In optimization iteration process, the basic sequence  $\mathbf{F}_{00}$  is denoted as the minimum of relevant objective function.

# ELITE MULTI-PARENT CROSSOVER ALGORITHM IMPROVED BY MIXED DISCRETE VARIABLES

#### Elite multi-parent crossover algorithm

Optimization problem is denoted as:

min  $f(\mathbf{x}), \mathbf{x} \in D, D = \{ \mathbf{x} \in S; g_k(\mathbf{x}) \le 0, k = 1, 2, \cdots, q, h_j(\mathbf{x}) = 0, j = 1, 2, \cdots, p \}$ 

where  $S \subset \mathbb{R}^n$  is search space;  $l_i \leq x_i \leq u_i (i = 1, 2, \dots, n)$ ; f is objective function; n the number of variables; D the feasible set of points;  $g_k$  constraint function; q the number of constraints. It is assumed that

$$H(\mathbf{x}) = h(\mathbf{x}) \left(\sum_{k=1}^{q} \mu(\phi_k(\mathbf{x}))\phi_k(\mathbf{x})^{\delta(\phi_k(\mathbf{x}))} + \sum_{j=1}^{p} \mu(\phi_j(\mathbf{x}))\phi_j(\mathbf{x})^{\delta(\phi_j(\mathbf{x}))}\right)$$
(4)

where  $\phi_k(\mathbf{x}) = \max\{0, g_k(\mathbf{x})\}, \phi_j(\mathbf{x}) = |h_j(\mathbf{x})|; h \text{ is punishment level; } h(\bullet), \mu(\bullet) \text{ and } \delta(\bullet)$ depend on specific problems<sup>[12]</sup>.

a is the bigger positive number, and the logistic function<sup>[13]</sup> is defined as:

$$better(\mathbf{x}_1, \mathbf{x}_2) = \begin{cases} true, if & H(\mathbf{x}_1) < H(\mathbf{x}_2) \\ false, if & H(\mathbf{x}_1) > H(\mathbf{x}_2) \\ true, if & (H(\mathbf{x}_1) = H(\mathbf{x}_2)) \land (f(\mathbf{x}_1) \le f(\mathbf{x}_2)) \\ false, if & (H(\mathbf{x}_1) = H(\mathbf{x}_2)) \land (f(\mathbf{x}_1) > f(\mathbf{x}_2)) \end{cases}$$

If *better*( $\mathbf{x}_1, \mathbf{x}_2$ ) is true, then  $\mathbf{x}_1$  will be better than  $\mathbf{x}_2$ ; otherwise,  $\mathbf{x}_2$  better than  $\mathbf{x}_1$ .

The algorithm is described as follows.

Step 1: Initial group, denoted as  $P_0 = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N\}$ , is generated in search space S, where N is the number of initial group and t = 0.

Step 2:  $P_t$  is sequenced from good to bad according to logistic function  $better(\mathbf{x}_1, \mathbf{x}_2)$ , and it is denoted that  $P_0 = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N\}$ , where  $\mathbf{x}_1$  is the best individual and  $\mathbf{x}_N$  the worst.

Step 3: Turn to Step 5 if  $better(\mathbf{x}_{worst}, \mathbf{x}_{best})$  is real (the best and worst individuals are the same).

Step 4:  $K(K \le M)$  best individuals  $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_K$  are selected from  $P_t$ . M - Kindividuals  $\mathbf{x}_{K+1}, \mathbf{x}_{K+2}, \dots, \mathbf{x}_M$  are randomly selected from the rest of  $P_t$   $(\mathbf{x}_{K+1}, \mathbf{x}_{K+2}, \dots, \mathbf{x}_N)$ . The subspace, formed by  $M(M \le N)$  individuals, is  $V = \left\{ \mathbf{x} \mid \mathbf{x} \in S, \mathbf{x} = \sum_{i=1}^{M} a_i \mathbf{x}_i^{'} \right\}$ , where  $a_i$ 

satisfies  $\sum_{i=1}^{M} a_i = 1$  and  $-0.5 \le a_i \le 1.5$ . *L* points are randomly selected in subspace *V* to achieve *L* new individuals where the best point is denoted as  $\overline{\mathbf{x}}$ . If  $better(\overline{\mathbf{x}}, \mathbf{x}_{worst})$  is real, then

 $\mathbf{x}_{worst}$  will be substituted by  $\overline{\mathbf{x}}$  to form a new individual  $P_{t+1}$ ; otherwise,  $P_{t+1} = P_t$ . Set t = t+1 and turn to Step 2.

Step 5: The best solution, where the whole function values are zero, is exported and the algorithm is over.

In Step 4, K elites in group  $P_t$  are made to be part of subspace V to make full use of better solution information and promote solution optimization. This is suitable to unimodal function optimization problem <sup>[14]</sup>. Numerical results indicate that the convergence is accelerated by this elite perversion strategy. K should be rationally selected. Solution information can be better used by a large K. However, a large K will cause a small degree of freedom in subspace V and local optimum for multimodal function. Selection of K is relevant to M. L individuals are selected in subspace V to effectively make use of information. For optimization problems, it is assumed that K = M/2 (or slightly less than M/2) and L = M, where M is equal to 1-3 times the number of variables.

### Engineering treatment of design variables

Discrete treatment of discrete design variables <sup>[1]</sup>

In elite multi-parent crossover algorithm, the new individual is a continuous variable. So, variables should be conducted with discrete treatment after updating of the group. Discrete method of integer design variables is similar with that of non-equidistant discrete variables. The difference is that the value of integer design variable is non-negative integral between the upper and lower limits.

#### Engineering treatment of continuous design variables

Although being continuous in form, some design variables are limited by precision of machine manufacturing and design specification in engineering optimization design. Floating point or double precision real is calculated based on programming language in optimization design. After that, data treatment is carried out according to decimal place required by practical engineering. The design obtained may not be the optimal solution and satisfy the constraints. Therefore, the data should be accurate to the given decimal place according to practical requirements.

#### Procedure

The above algorithm is improved to build dynamic penalty function. Using mixed variable treatment [15], the work develops elite multi-parent crossover optimization algorithm procedure MOPTDEMPCOA based on relative degree of grey incidence to solve high-dimensional multi-objective optimization design.

#### APPLICATION EXAMPLE

*Case 1:* In order to verify the method in the work, the valve spring given by Reference [16] is conducted with multi-objective optimization. The spring should have the lightest weight, the minimum height of freedom and the largest natural vibration frequency. Its optimization model is as follows:

$$\mathbf{x} = [x_1, x_2, x_3]^{\mathrm{T}}$$
  

$$f_1(\mathbf{x}) = x_1^2 x_2(x_3 + 1.8) \to \min$$
  

$$f_2(\mathbf{x}) = x_1(x_3 + 1.3) \to \min$$
  

$$f_3(\mathbf{x}) = 3.56 \times 10^5 \frac{x_1}{x_2^2 x_3} \to \max$$
  
S.t.  

$$g_1(\mathbf{x}) = 6.5 - |x_2 / x_1 - 9.5| \ge 0$$
  

$$g_2(\mathbf{x}) = 0.01 - |10^4 x_1^4 x_2^{-3} x_3^{-1} / 47 - 1| \ge 0$$
  

$$g_3(\mathbf{x}) = 405 - 2771 x_1^{-2.86} x_2^{0.86} \ge 0$$
  

$$g_4(\mathbf{x}) = 3.74286 - \frac{(x_3 + 1.3)x_1 + 18.25}{x_2} \ge 0$$

 $g_{5}(\mathbf{x}) = 3.56 \times 10^{5} x_{1} x_{2}^{-2} x_{3}^{-1} - 250 \ge 0$   $g_{6}(\mathbf{x}) = x_{3} - 3 \ge 0$   $g_{7}(\mathbf{x}) = x_{2} - 30 \ge 0$   $g_{8}(\mathbf{x}) = 60 - x_{2} \ge 0$   $g_{9}(\mathbf{x}) = x_{1} - 2.5 \ge 0$  $g_{10}(\mathbf{x}) = 9.5 - x_{1} \ge 0$ 

where  $f_1(\mathbf{x})$ ,  $f_2(\mathbf{x})$  and  $f_3(\mathbf{x})$  are objective functions of spring weight, height and natural vibration frequency;  $x_1$ ,  $x_2$  and  $x_3$  diameter, medium diameter and effective coil number of spring wire. The discrete variables are conducted according to national standard. MOPTDEMPCOA optimization operation by 3 times achieves Solution 1 in Table 1. The solution in the work is better than that in Reference [16].

Design Parameters	<i>x</i> <sub>1</sub>	<i>x</i> <sub>2</sub>	<i>x</i> <sub>3</sub>	$f_1(\mathbf{x})_{(\min)}$	$f_2(\mathbf{x})_{(\min)}$	$f_3(\mathbf{x})_{(\max)}$
Solution of Reference [16]	5.5	31	6.5	7783	42.9	313.5
Solution 1 in The Work	5.5	34	5	6993.8	34.65	338.75433
Solution 2 in The Work	6	45	3	7776	25.8	351.60

**Table 1:** Optimization and comparison of design parameters

Case 2: Optimization design of gear reducer

Optimization model of a certain single stage reducer in Reference [12] is denoted as:

min	$f(x) = 0.78539815(4.75x1x2^{2}x3^{2} + 85x1x2x3^{2} - 85x1x3^{2} + 0.92x1x6^{2} - x1x5^{2}$
	$+ 0.8x1x2x3x6 - 1.6x1x3x6 + x4x5^{2} + x4x6^{2} + 32x6^{2} + 28x5^{2})$
s.t.	$g_1(x) = 17 - x2 \le 0$
5.1.	$g_2(x) = x2x3 - 30 \le 0$
	$g_7(\mathbf{x}) = x_5 - 15 \le 0$
	$g_8(\mathbf{x}) = 13 - x_6 \le 0$
	$g_9(\mathbf{x}) = x6 - 20 \le 0$
	$g_{10}(\mathbf{x}) = x_1 + 0.5x_6 - x_4 + 4 \le 0$
	$g_{11}(\mathbf{x}) = \frac{43854}{x_2 x_3 \sqrt{x_1}} - 855 \le 0$
	$g_{12}(\mathbf{x}) = \frac{7098}{x_1 x_2 x_3^2(0.\ 169 + 0.\ 006666 x_2 - 0.\ 0000854 x_2^2)} - 261 \le 0$
$\mathcal{S}_{12}(\mathbf{x})$	$x_1 x_2 x_3^2 (0.169 + 0.006666 x_2 - 0.0000854 x_2^2)$
	$g_{13}(\mathbf{x}) = \frac{7098}{x_1 x_2 x_3^2 (0.\ 2824 + 0.\ 00177 x_2 - 0.\ 0000394 x_2^2)} - 213 \le 0$
	$g_{14}(\mathbf{x}) = \frac{0.01233x_4^3}{x_1x_3x_5^4} - 0.003x_4 - 261 \le 0$
	$g_{15}(\mathbf{x}) = 29050 \frac{29050 x_4}{x_2 x_3 x_5^3} \sqrt{1 + \frac{0.29709 x_2^2 x_3^2}{x_4^2}} - 55 \le 0$
	$g_{16}(\mathbf{x}) = \frac{29050x_4}{x_2 x_3 x_6^3} \sqrt{1 + \frac{7.42727 x_2^2 x_3^2}{x_4^2}} - 55 \le 0$

In the model, objective function f(x) is volume of reducer  $({^{Cm}}^3).x_1, x_2, x_5$  and  $x_6$  are integer variables, where  $x_1$  is width of gear  $({^{Cm}}); x_2$  the number of gear teeth;  $x_5$  and  $x_6$  are shaft diameters of two gears  $({^{Cm}}).x_3$  is a discrete variable denoting module of gear;  $x_4$  a continuous variable denoting width of gear case and accurate to two decimal places. To increase robustness of the result, the change rate of objective function to each variable should be the minimum. Optimization of the gear is transformed into multi-objective optimization problem.

$$\min \mathbf{F}(\mathbf{x}) = [f(\mathbf{x}) \quad \left| \frac{\partial f(\mathbf{x})}{\partial x_1} \right|, \quad \left| \frac{\partial f(\mathbf{x})}{\partial x_2} \right|, \quad \left| \frac{\partial f(\mathbf{x})}{\partial x_3} \right|, \quad \left| \frac{\partial f(\mathbf{x})}{\partial x_4} \right|, \quad \left| \frac{\partial f(\mathbf{x})}{\partial x_5} \right|, \quad \left| \frac{\partial f(\mathbf{x})}{\partial x_6} \right|]^{\mathrm{T}}$$

The constraint is the same with constraint function of single objective optimization function  $f(\mathbf{x})$ . Table 2 and 3 show the results achieved by the method in the work. The optimization solution is better than that by the present method.

Table 2: Optimization design parameters of reducer						
Design Parameters	$x_{1/cm}$	<i>x</i> <sub>2</sub>	<i>x</i> <sub>3</sub>	$x_4$ /cm	$x_{5/cm}$	$x_{6/cm}$
Solution in Reference [1]	13	24	0.60	23.75	10	13
Solution 1 in The Work	11	24	0.65	24.49	10	13
Solution 2 in The Work	4	129	0.2	17	10	13

 Table 3: Objective function value of reducer optimization

Objective Function	f(x) / <b>cm</b> <sup>3</sup>	$\frac{\partial f}{\partial x_1}$	$\frac{\partial f}{\partial x_2}$	$\frac{\partial f}{\partial x_3}$	$\frac{\partial f}{\partial x_4}$	$\frac{\partial f}{\partial x_5}$	$\frac{\partial f}{\partial x_6}$
Solution in Reference [1]	30675	1478	1214	59811	211	609	1490
Solution 1 in The Work	30507	1716	1200	54662	211	651	1459
Solution 2 in The Work	22342	3076	171	117152	211	644	1139

## CONCLUSIONS

The work proposed a high-dimensional multi-objective optimization design method. The method applied relative degree of grey incidence to transform high-dimensional multi-objective optimization problem into single objective one. After that, the single objective problem was solved by elite parent crossover optimization algorithm. The dynamic penalty function and elite preservation strategy were introduced to add parent selection pressure and accelerate algorithm convergence. The work developed elite multi-parent crossover optimization algorithm software MOPTDEMPCOA based on high-dimensional multi-objective optimization problem of mixed discrete variables. This software can rationally deal with value of mixed discrete variables in optimization problems. This method, with good global convergence and reliable operation, can be widely and effectively used to solve high-dimensional multi-objective optimization and robust design problems of mixed discrete variables.

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